BBC RD 1988/13

Research Department Report

Investigation of mutual interference between digitally modulated signals

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INVESTIGATION OF MUTUAL INTERFERENCE BETWEEN DIGITALLY MODULATED SIGNALS

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Summary

This Report considers the effects of co-channel and adjacent-channel interference on a QPSK modulation system. From previous theoretical studies it identifies an upper and lower error bound. It examines practically how useful these bounds are at predicting the performance of QPSK at various levels of noise and interference.

The two error bounds gave a good indication of the error performance for QPSK with single and multiple interferers. The results should also be applicable to other types of digital phase modulation.

Issued under the Authority of

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Head of Research Department

Research Department, Engineering Division,
BRITISH BROADCASTING CORPORATION

(RA-241)

December 1988

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1. INTRODUCTION

There is much interest being shown in the development of digital broadcast systems. Direct radio broadcasting by satellite[1] and high definition television[2] are two services for which a digital modulation system is being considered. For both these services there are as yet no frequency allocations. Broadcasters will only be allotted frequencies if they show that they can use them efficiently to provide an international service. This means that many services will have to operate in limited spectrum space, giving rise to adjacent-channel and co-channel interference. The number of services which can use a frequency band will depend on the noise and interference performance of the modulation system which is chosen.

To plan these services efficiently the performance of different modulation systems with noise and interference needs to be evaluated.

This Report considers the performance of quaternary phase shift keying (QPSK) with single and multiple digital interferers. It starts by considering some of the theoretical work that has already been done on PSK modulation with sinusoidal interference and noise. From this theory it is possible to pick out some important and useful results which can then be verified experimentally. For the practical work a system operating at 728 kbit/s was used. The digital interferers operated at the same bit rate and had similar spectral shapes to the wanted signal. This is consistent with broadcasting, where interferers are generally very similar to the main signal. Although the bit rate is quite low, it is representative of sound broadcasting. For television, the results can be scaled for the larger bit rate and the wider bandwidth needed.

2. THEORY OF INTERFERENCE WITH RANDOM NOISE

2.1 General

In the absence of random noise and distortion, a digital channel should be error free. An interfering signal on its own will only start to produce errors when it reaches a certain level. A simple way of determining this level is to consider the vector sum of the wanted signal and the interference. For an m-phase system, the resultant's phase must not have changed by more than π/m radians. For QPSK with one sinusoidal interferer, the carrier-to-interference ratio (C/I) must exceed 3 dB* for error-free reception. This argument can be extended to any number of sinusoids.

Random noise will always be present with any interference. In this case the analysis is complicated and no easy solution exists. In simple terms the interfering signal, even at fairly low levels, will make the PSK decoder more susceptible to any random noise in the channel. This will produce a degradation in performance which is dependent on both the level and the nature of the interference.

Rosenbaum[3] and Prabhu[4] have tackled this problem and have produced theoretical results for sinusoidal interference. Rosenbaum provides a solution for the case of a single sinusoidal interferer. Prabhu's results are more general in that they can be extended to any number of sinusoidal interferers. He asserts that the error performance is dependent on the level of interference, the number of interferers and the distribution of the interference power. If the interference power is concentrated into a single interferer the error rate will be at its lowest. Conversely, when the power is distributed equally among N interferers, the error rate will be at a maximum. As N increases, the maximum error rate also increases. For large N (greater than 50) the probability density function approaches that of gaussian noise. This will give the worst performance.

These results give two very useful bounds for determining error performance with gaussian noise and interference. The lower bound is given by considering the interference power to be concentrated into one sinusoidal interferer. The upper bound is given by treating the interference as gaussian noise of the same average power.

2.2 Solution of Prabhu's equation for several interferers

The solution of Prabhu's equation becomes more difficult as the number of interferers increases. He shows that the bit-error ratio (BER) for BPSK (binary phase shift keying) and QPSK suffering random noise and interference is given by Equation 1.

* In practice, receiver imperfection will always mean this is an optimistic bound.

$$\text{BER} \doteq \frac{1}{2}\,\text{erfc}\left(\rho \sin \frac{\pi}{m}\right) +$$

$$\frac{1}{(\pi)^{1/2}} \exp\left(-\rho^2 \sin^2 \frac{\pi}{m}\right) \cdot$$

$$\sum_{n=1}^{\infty} \frac{H_{2n-1}\left(\rho \sin \frac{\pi}{m}\right)}{(2n)!} \rho^{2n}\, \mu_{2n} \qquad (1)$$

where $\quad \text{erfc}(u) = \dfrac{2}{\sqrt{\pi}} \displaystyle\int_u^{\infty} e^{-v^2}\, dv$

H_n = Hermite function of order n

m = Number of phases (m=2 or m=4)

ρ^2 = Carrier-to-noise ratio (linear)

μ_{2n} = 2^{nth} central moment of random variable η

$$\eta = \sum_{j=1}^{K} R_j \cos \lambda_j$$

K = Number of interferers

R_j = Carrier to j^{th} interferer amplitude ratio (linear)

λ_j = Angle of j^{th} interferer to carrier

The calculation of the error function and Hermite polynomials is trivial. The error function can be calculated quickly to any accuracy using Tchebyshev expansions[5] and the Hermite polynomial can be calculated using a recurrence formula[6]. Problems arise when calculating μ_{2n}.

For one interferer the solution is easy to calculate and given by

$$\mu_{2n} = R^{2n} \frac{(2n)!}{2^{2n}(n!)^2} \qquad (2)$$

For more interferers there is no simple solution. Prabhu gives two methods for calculating the second moments, namely:

$$\mu_{2n} = \frac{1}{(2\pi)^K} \int_0^{2\pi} d\theta_1 \int_0^{2\pi} d\theta_2 \ldots \int_0^{2\pi} d\theta_K \cdot$$

$$\left[\sum_{j=1}^{K} R_j \cos \theta_j\right]^{2n} \qquad (3)$$

and

$$\mu_{2n} = \Omega^{2n}\Bigg(\underbrace{\frac{1}{2n+1}}_{a} + \underbrace{2\sum_{l=1}^{\infty} (-1)^{l+1} \left\{\left[\prod_{j=1}^{K} J_0\left(\frac{l\pi R_j}{\Omega}\right)\right] \cdot \sum_{k=1}^{n} (-1)^k \frac{(2n)!}{[2n-2k+1]!\,(l\pi)^{2k}}\right\}}_{b} \Bigg) \qquad (4)$$

where $\quad \Omega = \displaystyle\sum_{j=1}^{K} R_j$

and J_0 is the Bessel function of first kind, zero order.

Equation (3) has the following disadvantages:

(i) The integrand is of a high order, especially for large n. The numerical integration routine would have to calculate many terms to achieve accuracy for the higher orders.

(ii) The time to perform the calculation increases exponentially with the number of interferers i.e.

Time taken = (time for one interferer)^k where k is the number of interferers

Prabhu shows by using the multinomial theorem that the integration can be rewritten as a summation. This however, is just as difficult to calculate as the integeration for large values of n.

Equation (4) was also found to be impractical for intermediate* values of n. As n increases (i.e. the order of the moment increases) μ_{2n} tends to zero. This means that for large values of n, term b of the expression will be negative and its magnitude will tend to that of term a. Even for small values of n (less than twenty) the accuracy of the result is dependent on the least significant figures. This leads to large rounding errors, even when using functions accurate to seventeen significant figures. Unfortunately the rounding errors become significant before equation (1) has converged. The errors are worse for large values of carrier-to-noise ratio and an intermediate number of interferers. Because of these difficulties the BER was calculated from Prabhu's equation for only one sinusodidal interferer. This gives the theoretical lower error bounds given in Section 4.

* Here intermediate means any value of n between 4 and 50.

3. EXPERIMENTS

3.1 Experimental configuration

The experimental configuration is shown in Fig. 1. It allows random noise and interference (sinusoidal or digital) to be added to the main (or wanted) signal.

The main signal was provided by a digital modulator programmed to produce differentially-coded QPSK. Its data were generated by a pseudo-random binary sequence (PRBS) generator at 728 kbit/s.

Three digital interferers were available. The interference was either QPSK or vestigial-sideband binary phase-shift keying (VSB-2PSK)[7]. Again simple PRBS generators provided the data. Each generator was similar to that used for the main signal, but was programmed to give a different sequence. This was done to reduce any correlation between the main and interfering signals. The outputs of the digital modulators were added using a resistive network to give the digital interference signal. Each digital modulator could be turned off, allowing the digital interference to be generated from either one, two or three interferers.

A power meter was used to measure all signal and interference levels. A gaussian bandpass filter, centre frequency 6.7 MHz and of known noise bandwidth, was used to calibrate the noise source.

The QPSK demodulator was connected to a sequence-correlating error checker which gave a direct reading of the bit-error ratio.

3.2 Description of experiments

The following measurements were made:

(i) Effect of gaussian noise.

The error performance of the demodulator was measured over a range of carrier-to-noise (C/N) ratios.

(ii) Effect of sinusoidal interference with gaussian noise.

The error performance of the demodulator was measured with a sinusoidal interferer at carrier frequency (6.552 MHz) and gaussian noise added to the wanted signal.

(iii) Effect of offset sinusoidal interference with gaussian noise.

Initially the C/I ratio was set to 15 dB with no frequency offset, i.e. the interfering sinusoid was at carrier frequency. The C/N ratio was adjusted to give a bit-error ratio of 10^{-3}. For each frequency offset the C/I ratio was measured at which the bit-error ratio was 10^{-3}.

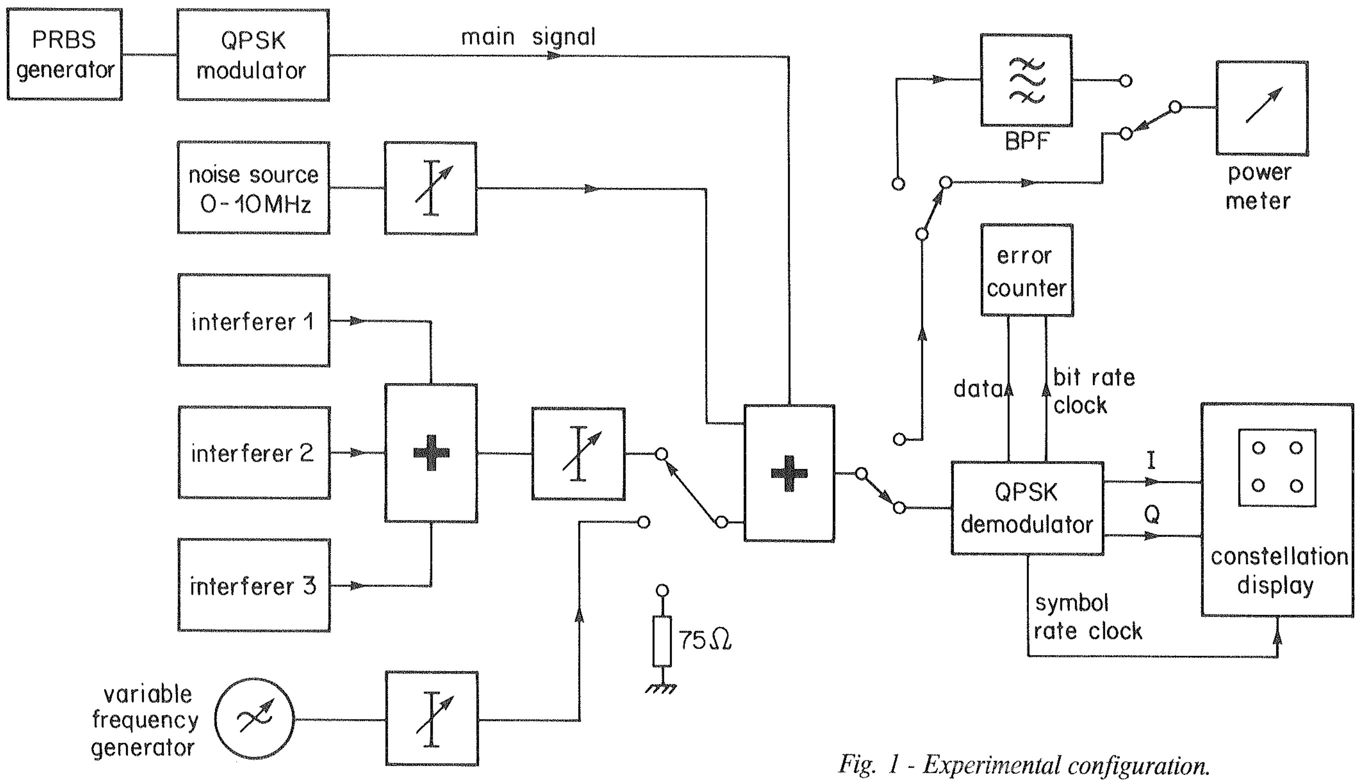


Fig. 1 - Experimental configuration.

(iv) Effect of digital interference with gaussian noise.

The error performance was measured over a range of C/I ratios and C/N ratios using one digital interferer as the interference signal. A further two sets of measurements were made using two and three interferers. For each experiment the interference power was distributed equally amongst the interferers.

(v) Effect of digital interferer with offset carrier.

The same procedure given in (iii) was used except a digital interferer, whose carrier frequency could be varied, replaced the sinusoidal interferer.

During each of these experiments the eye diagram and constellation display were observed.

4. RESULTS

Fig. 2 shows computer simulated QPSK patterns with ideal channel filtering. The filtering conformed to the Nyquist criterion and had 100 per cent cosine roll-off.

These diagrams serve as a reference for the results that follow. The constellation display (Fig. 2(c)), which is a display of the phase and the amplitude of the signal at the sampling instant, is particularly informative.

4.1 Performance of QPSK demodulator

Fig. 3 shows the measured patterns without any noise or interference added to the wanted signal. The eye-height of the I and Q channels is greater than 90 per cent and there is fairly good agreement with the simulated diagrams in Fig. 2. The slight thickening of the eye at the sampling points (or spreading of the sampling clusters in the constellation display) is caused by intersymbol interference. This is probably introduced by a slight asymmetry of the receive filter's response.

4.2 Noise performance

For a channel which complies with Nyquist's restrictions and uses a matched filter, the theoretical bit-error rate (BER) is given by

$$\text{BER} = \frac{1}{2} \operatorname{erfc} \sqrt{(E_b/N_o)}$$

where

E_b = Signal energy per bit incident on the receive filter, i.e. the mean power divided by the bit rate.

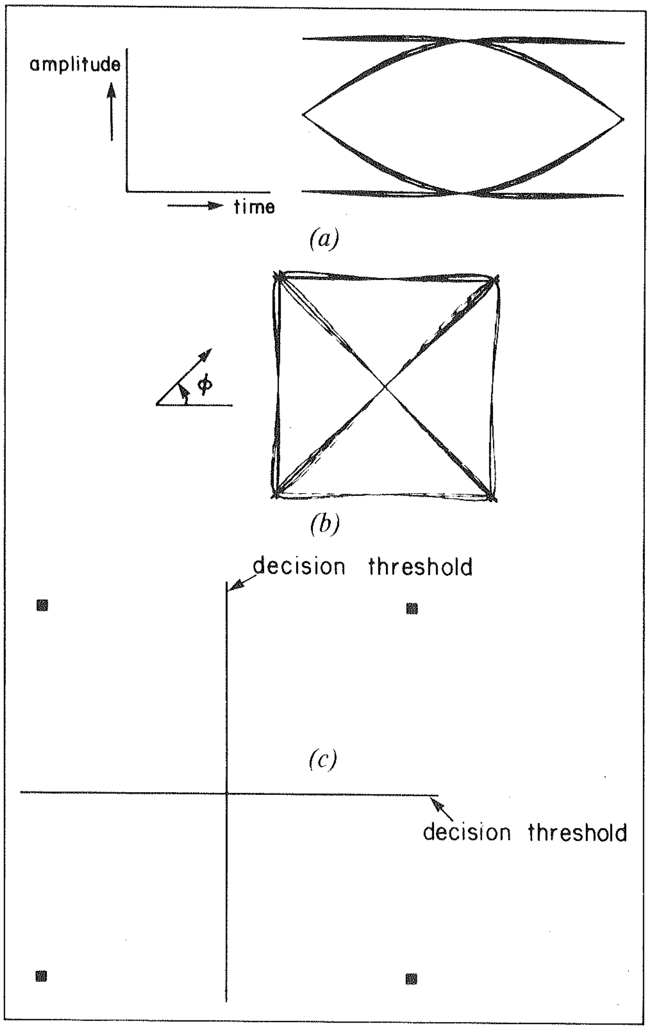
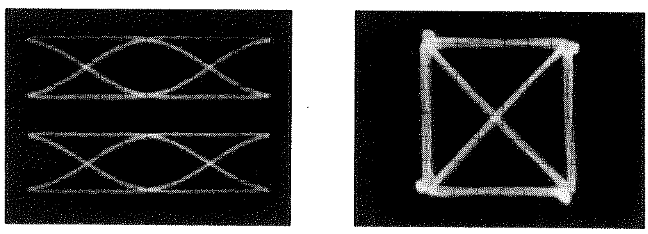
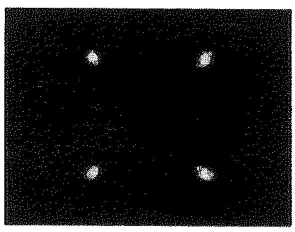


Fig. 2 - Ideal data patterns for 100% cosine roll-off filtering for QPSK:

(a) Eye diagram
(b) Phasor diagram
(c) Constellation display.



(a) (b)



(c)

Fig. 3 - Measured data patterns for QPSK:

(a) Eye diagram
(b) Phasor diagram
(c) Constellation display.

N_o = Single-sided noise power spectral density on the receive filter.

For QPSK, the carrier-to-noise ratio measured in a noise bandwidth of half the bit rate is given by:

$$C/N = E_b/N_o\,(\mathrm{dB}) + 3\ \mathrm{dB}$$

For differentially decoded QPSK, the BER is further modified to give

$$BER_D = 2BER\,(1 - BER)$$

where BER_D is the differentially decoded bit-error rate.

Fig. 4 shows the theoretical and measured error rates. There is good agreement, to within 0.2 dB, at bit-error ratios of 10^{-3} and 10^{-4}. For higher carrier-to-noise ratios the performance deteriorates slightly; this is probably due to the imperfections of the receive filter.

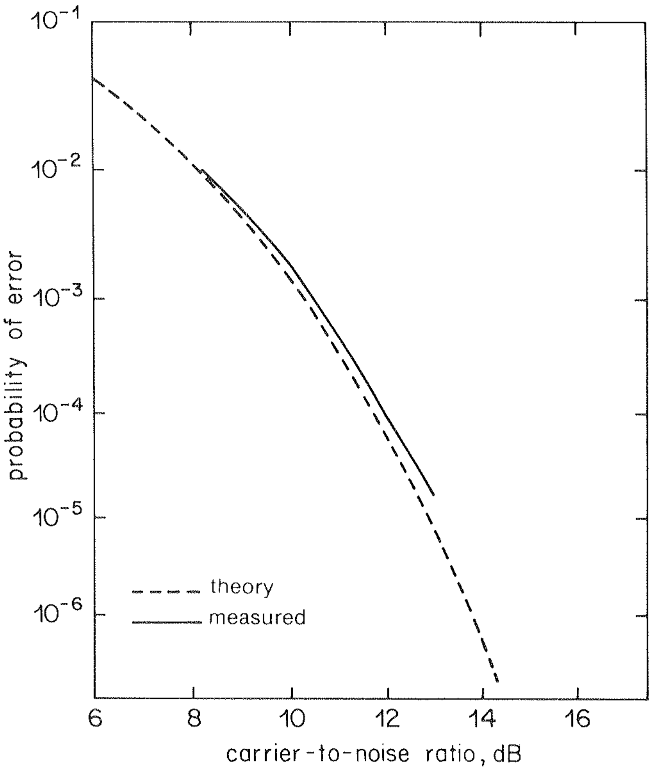
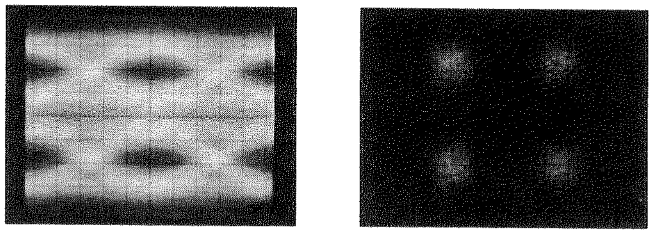


Fig. 4 - Error rate as a function of carrier-to-noise ratio for differentially encoded QPSK.

The effect of noise on the eye diagrams and constellation display can be seen in Fig. 5. The noise randomly perturbs the phase of the QPSK signal, giving the eye diagrams their blurred appearance. The constellation points have a 'fuzzy' look, showing that the phase of the carrier signal is now less clearly defined at the sampling points.



(a) (b)

Fig. 5 - Measured data patterns with random noise:

(a) Eye diagram C/N=10 dB
(b) Constellation display C/N=10 dB

4.3 Sinusoidal interference with random noise

4.3.1 Sinusoid with no offset

The theoretical performance of a digital system in the presence of a sinusoid with random Gaussian noise is discussed in Section 2.

Fig. 6 compares the ideal and measured performance over a range of carrier-to-interference ratios. There is generally good agreement, especially when considering that the error rate is now a function of two parameters. If we use the carrier-to-noise ratio alone to specify the results, then they are all within 1 dB of theory. Those for the 15 dB C/I and 20 dB C/I curves are all within 0.5 dB of theory.

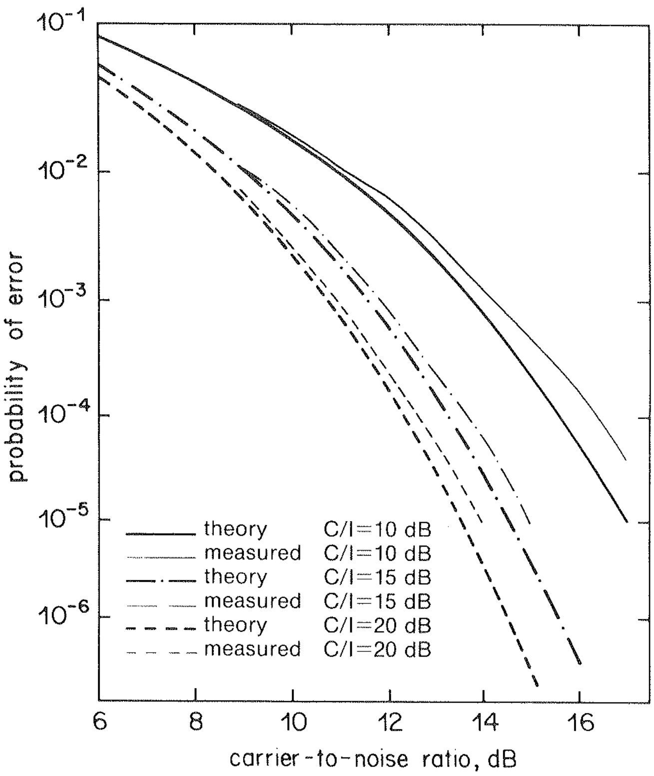
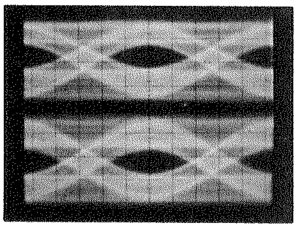
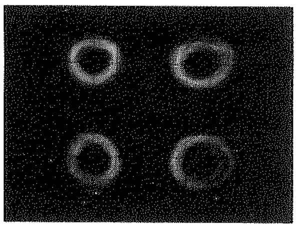


Fig. 6 - Error rate of QPSK with one sinusoidal interferer.

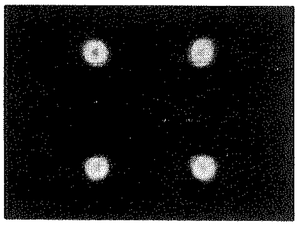
Fig. 7 shows the effect of sinusoidal interference on the eye diagrams and constellation display. The interferer sinusoidally perturbs the phase of the QPSK carrier. This causes the constellation of sampling points to rotate at the difference frequency of the signal and interferer. Hence we see circles on the constellation display, the diameters of which vary with the level of interference. Adding noise tends to blur the display, though at the level of carrier-to-noise ratio in Fig. 7(d), the circles can still be seen clearly.



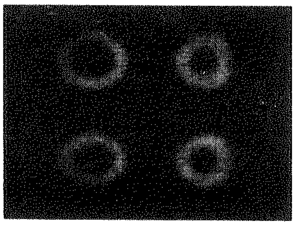
(a)



(b)



(c)



(d)

Fig. 7 - Measured data patterns with sinusoidal interferer:

(a) Eye diagram C/I=10 dB
(b) Constellation display C/I=10 dB
(c) Constellation display C/I=20 dB
(d) Constellation display C/I=10 dB, C/N=20 dB.

4.3.2 Sinusoid with frequency offset

The perfomance of the digital system with interference offset in frequency is shown in Fig. 8. The shape of the curve mirrors almost exactly the response of the cosine receive filter. This implies that the error performance with a sinusoid and noise is independent of frequency and only dependent on the amplitude or more correctly, the power of the interferer. This agrees with theory.

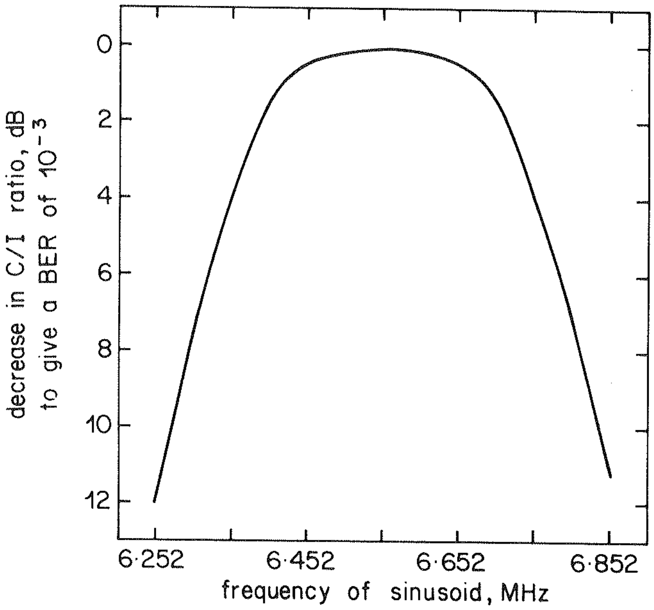


Fig. 8 - Change in C/I ratio needed to restore BER of 10^{-3} as a sinusoidal interferer is offset in frequency.

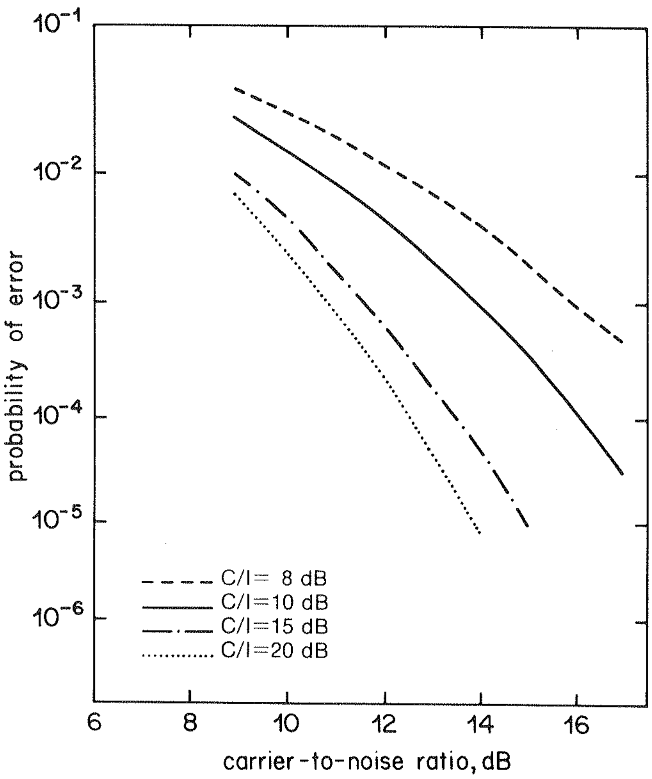


Fig. 9 - QPSK error rate with one digital interferer.

4.4 Digital interference with random noise

The results in this section apply to both QPSK interference and VSB-2PSK interference. The measured degradation in error performance and the distortions of the data patterns were very similar for both types of interference. The figures and results which follow show the effects of QPSK interference.

4.4.1 One digital interferer

Fig. 9 shows the performance of the digital system with one interferer over a range of carrier-to-interference ratios.

It is more interesting however to compare the effect of the sinusoidal and digital interference (see Fig. 10) and to note that there is close agreement. The error performance with the digital interferer is slightly better. The distortions of the eye diagram and constellation display caused by a single digital interferer, without added noise, (Fig. 11) are also very similar to that of sinusoidal interference (Fig. 7).

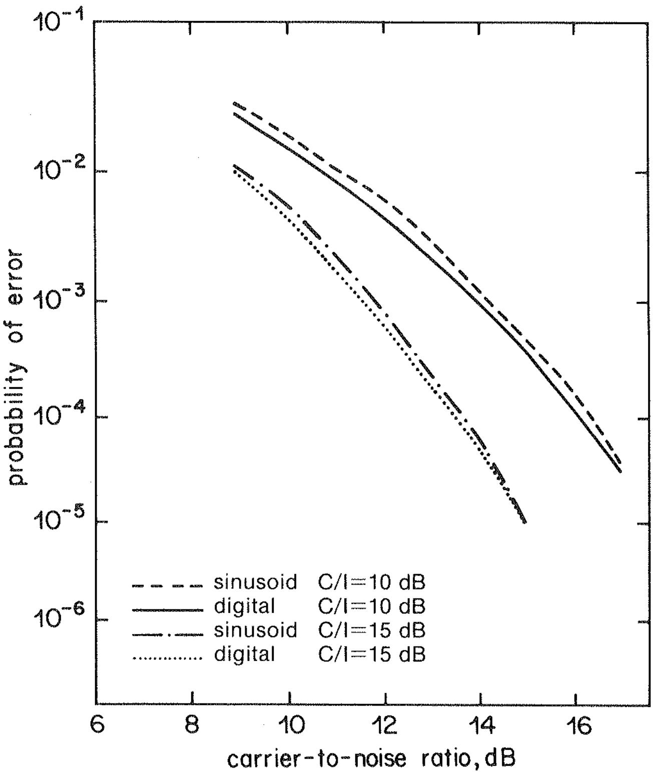
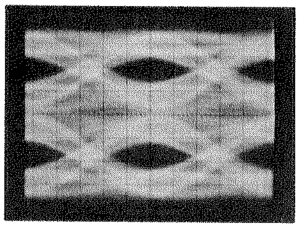
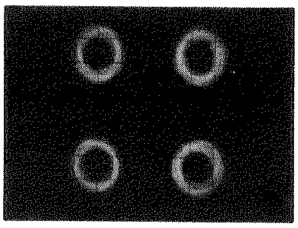


Fig. 10 - Comparison of interference by a sinusoid and a digital interferer.



(a)



(b)

Fig. 11 - Measured data patterns with one digital interferer:

(a) Eye diagram C/I=10 dB
(b) Constellation display C/I=10 dB

4.4.2 One digital interferer with frequency offset

The lower curve of Fig. 12 shows the error performance of QPSK with a digital interferer at various frequency offsets. The curve is much broader than that with the sinusoidal interferer (Fig. 8) because the spectrum of the digital interferer is much wider. We might postulate that the increase in BER caused by an interferer will be related to interference power seen at the output of the demodulator's cosine receive filter. Because we know the spectral shaping of the digital interferer, we can work out the interference power passing through the receive filter for different frequency offsets. The Appendix shows the calculations for a QPSK interferer. The upper curve of Fig. 12 shows the result for a cosine filter centred at 6.552 MHz. There is a close match between the theoretical interference power at the output of the receive filter and the measured degradation in performance.

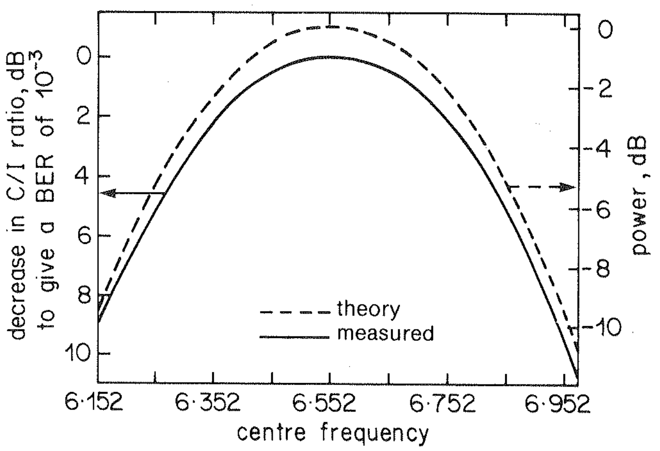


Fig. 12 - Effect of a frequency offset of a digital interferer:

upper trace: theoretical change in power passing through ideal cosine filter from a digital signal with a frequency offset.

lower trace: change in C/I ratio to restore a BER of 10^{-3} for an offset digital interferer.

4.4.3 More than one digital interferer

Fig. 13 compares the performance of the QPSK demodulator with one, two and three digital interferers. The graphs show the upper and lower bounds predicted by the theory. As predicted, the performance is worse for three interferers than it is for one, especially at higher carrier-to-noise ratios. Fig. 14 shows the effect on the eye diagram and constellation display with three digital interferers. It is interesting to note how 'noise-like' these displays are; in fact it is very difficult to tell them apart from Fig. 5. The most important difference is that the random noise causes errors, while the interference on its own does not.

5. DISCUSSION

The results shown in Fig. 10 suggest that a sinusoid at carrier frequency would make a good model for a co-channel QPSK (and VSB-2PSK) interferer.

The degradation in performance caused by interference is dependent on the C/I ratio seen at the output of the demodulator's receive filter. This might explain why a sinusoid at carrier frequency produces a poorer performance than a co-channel digital interferer (Fig. 10). Neglecting insertion loss, the sinusoid will pass through the receive filter without any attenuation. A digital interferer, however, having a broader spectrum, will be shaped by the filter in the same way as the wanted QPSK signal. In fact, the receive filter attenuates the power of the digital interferer by just

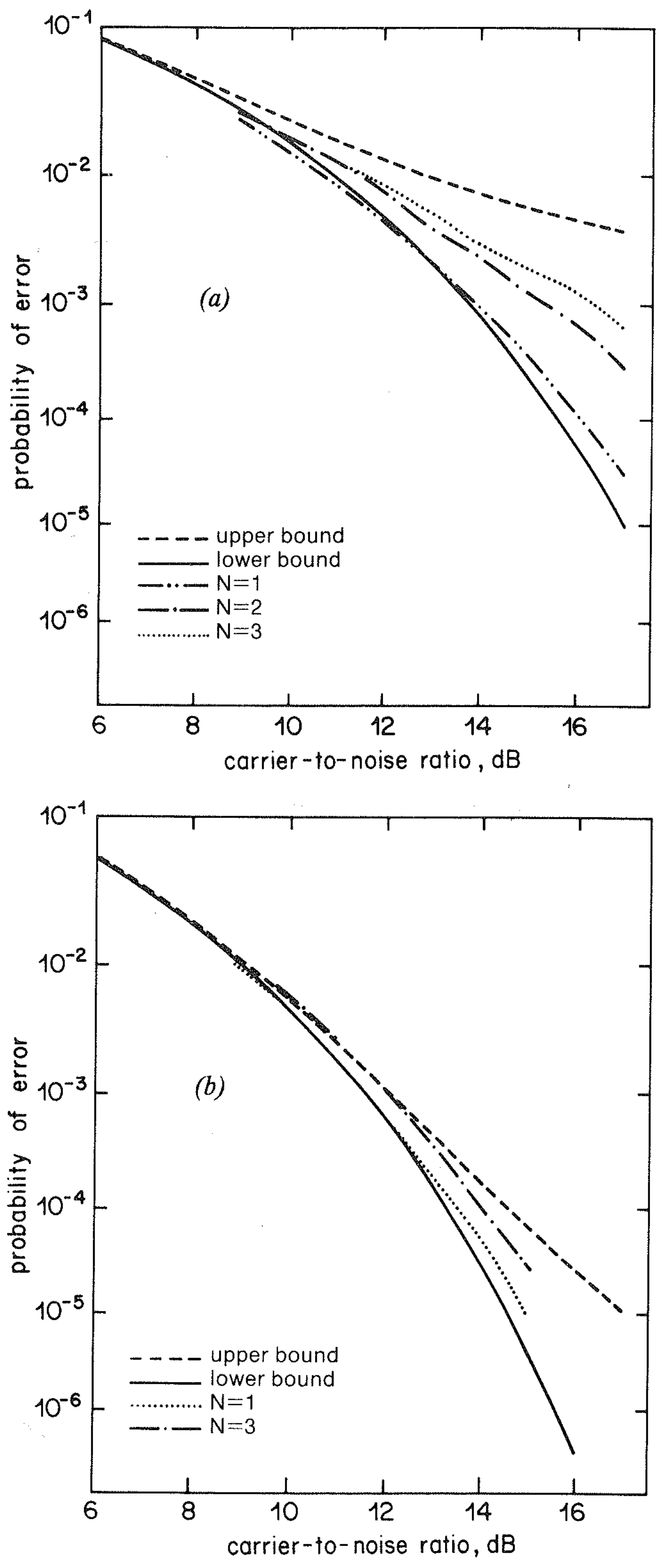
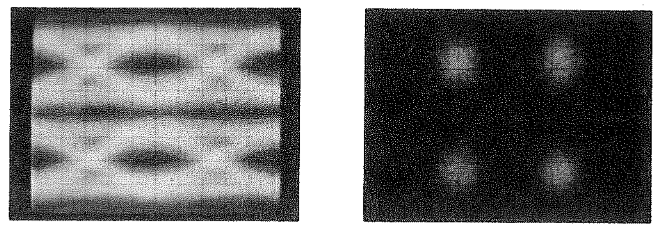


Fig. 13 - QPSK error rate with N *digital interferers:*

(a) C/I=10 dB
(b) C/I=15 dB.

over 1 dB. For a given interference power this will result in a sinusoid giving a lower C/I ratio at the filter's output than a digital interferer.

We can develop this idea further to deal with adjacent-channel interference. For an offset sinusoid the change in C/I ratio is simply given by the receive filter's response at the offset frequency. The problem is not quite so easily solved for a digital interferer. Again we need to estimate the change in C/I ratio at the filter's output caused by the frequency offset. The Appendix shows a method of calculating this.



(a) (b)

Fig. 14 - Measured data patterns for three digital interferers:

(a) Eye diagram C/I=10 dB
(b) Constellation display C/I=10 dB

The lower and upper bounds are useful when we have to consider more than one interferer (or dominant interferer). They are easy to calculate and seem to fit in well with the measured results (Fig. 13). It can be seen that as the number of interferers increases, the error rate becomes greater and approaches the upper bound. The two bounds do not take into account any implementation loss in the equipment. None the less they are a good approximation.

The difference between the upper and lower bounds gets larger as the carrier-to-noise ratio increases. In general, the upper bound should be used for planning a reliable service. For example, in digital sound broadcasting the failure point of a service is typically at a BER of 10^{-3}. At this error rate the difference between the bounds is small, even at low C/I ratios. Using the upper bound in this instance will not effect greatly the efficient planning of a service. For digital television the failure point usually occurs at a lower error ratio, typically at a BER of 10^{-6}. At this error rate the difference between the two bounds can be large and planning a service efficiently with the upper bound may be difficult. To improve efficiency the lower bound may be applied to service areas where the source of interference comes from either a single interferer or from a single dominant interferer.

For an example we shall assume a service area with a hexagonal structure. Ref. 1 describes a simple planning scheme using this assumption, and gives some indication of the likely levels of the adjacent-channel interference (ACI) and the co-channel interference (CCI), which may be present. Assume that the modulation system will be QPSK, using 100% cosine roll-off for the channel filtering. The data rate will be

728 kbit/s and the channel spacing will be 550 kHz. From Ref. 1 we may assume:

$$\begin{aligned} \text{carrier-to-noise ratio} &= 12.0 \text{ dB} \\ \text{carrier-to-CCI ratio} &= 16.1 \text{ dB} \\ \text{carrier-to-ACI ratio} &= -1.1 \text{ dB} \end{aligned}$$

Using the result in the Appendix with a channel spacing of 550 kHz, the carrier-to-ACI ratio is modified to give:

effective carrier-to-ACI ratio
after receive filter = 20.6 dB

Therefore

effective C/I ratio = 14.8 dB

By using the upper error bound, the interference power will, at worst, degrade the C/N ratio by 1.8 dB.

6. CONCLUSIONS AND RECOMMENDATION

A study has been made to determine the amount of degradation random noise and interference caused to a digitally modulated signal. This study included sinusoidal and digitally modulated interference at both co-channel and offset frequencies.

For each type of interference, the distortions of the eye diagram and the constellation display are shown. The BER measurements for sinusoidal interference are within 1 dB of theory.

By considering the theory of sinusoidal interferers it is possible to estimate an upper and a lower bound on the error performance for a digital channel. For a single interferer, or a dominant interferer, the lower bound gives a good approximation. For many interferers of similar levels, the error performance will tend to the upper bound. The upper bound is equivalent to the performance obtained when a noise power equal to the total interference power is added.

If we allow for the channel filtering in the demodulator then these results apply to adjacent-channel interference as well.

For all the experiments described in this Report the constellation display illustrated the effects of noise and interference very clearly. It is a good representation of QPSK as it combines the information of the I and Q eye diagrams in a compact and simple way. It also allows us to estimate the level of noise or interference quite easily, and to distinguish between noise, interference and distortion.

It is recommended that the constellation display be considered to assess and to monitor the performance of all digital systems when used in service.

7. REFERENCES

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APPENDIX

Relationship Between the Received Interference Power and the Frequency Offset of an Interferer.

This appendix derives an expression relating the frequency offset of an interferer to the power passing through the receive filter. The filter's response and the spectrum of the interferer are assumed to be the same and both have the square root of 100 per cent cosine roll-off shaping. (See Fig. A.1).

The filter has a centre frequency of f_c Hz and a 3 dB bandwidth of f_b Hz. The interferer is displaced from f_c by f_d.

$$\Delta\phi = \frac{\pi f_d}{2 f_b}$$

$$\theta = \frac{\pi(f-f_c)}{2 f_b} \qquad f-2f_b < f < f+2f_b$$

$$E(\Delta\phi) = \int_{\Delta\phi-\pi/2}^{\pi/2} [\ \cos(\theta)\cos(\theta-\Delta\phi)\ ]^2 \, d\theta$$

Which on simplification leads to

$$E(\Delta\phi) = \frac{1}{4}(\pi-\Delta\phi)\left[\frac{1}{2}+\cos^2(\Delta\phi)\right]+\frac{3}{16}\sin(2\Delta\phi)$$

Using zero offset ($\Delta\phi = 0$) as a reference we get

$$\frac{E(0)}{E(\Delta\phi)} = \frac{6\pi}{(\pi-\Delta\phi)\ (2+4\cos^2(\Delta\phi))\ +3\sin(2\Delta\phi)}$$

This gives the desired relationship between the power received with a frequency offset and that without.

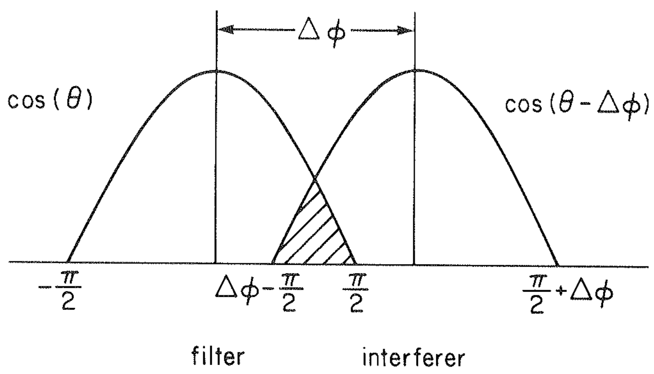

Fig. A.1 - Example of a QPSK spectrum overlapping the receive filter's response.

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